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Kinking out of a Mixed Mode Interface Crack

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ABSTRACT

In this study we focus on the mixed mode fracture toughness and the kink angle of an interface crack. We measure residual stress and perform mixed mode fracture tests for three types of interface crack. Each mixed mode fracture toughness including residual stress is successfully described by stress intensity factors K_I and K_{II} for each interface crack. The kink angle of each interface crack also can be expected by the stress intensity factors using the modified maximum hoop stress criterion.

1. STRESS INTENSITY FACTORS AND CRACK KINKING OF AN INTERFACE CRACK

1. 1 Stress Intensity Factors of an Interface Crack

The asymptotic solution of the stress distribution around an interface crack as shown in Fig. 1 was proposed by Erdogan [1]. The stress along the *x*-axis is shown as

$$\left(\sigma_{yy} + i\sigma_{xy}\right)_{\theta=0} = \frac{K_I + iK_{II}}{\sqrt{2\pi r}}\left(\frac{r}{l_k}\right)^{i\varepsilon} \tag{1}$$

$$\alpha = \frac{\mu_1(\kappa_2+1) - \mu_2(\kappa_1+1)}{\mu_1(\kappa_2+1) + \mu_2(\kappa_1+1)}, \quad \beta = \frac{\mu_1(\kappa_2-1) - \mu_2(\kappa_1-1)}{\mu_1(\kappa_2+1) + \mu_2(\kappa_1+1)} \tag{2}$$

$$\varepsilon = \left(1/2\pi\right)\ln\left(1 - \beta/1 + \beta\right) \tag{3}$$

$$\begin{cases} \kappa_i = 3 - 4\nu_i & (Plane\ strain) \\ \kappa_i = \left(3 - \nu_i\right)/\left(1 + \nu_i\right) & (Plane\ stress) \end{cases} \tag{4}$$

where α and β are Dundurs's parameters [2], ε is the bimaterial constant, and μ_1, μ_2, ν_1 and ν_2 are shear moduli and Poisson's ratios for respective materials. σ_{yy} and σ_{xy} are stresses, K_I and K_{II} are the stress intensity factors (SIF) for respective mode and i is the complex number ($i^2 = -1$). l is an arbitary characteristic number. Argument of the stress in Eq. (1) is shown as

$$\arg(\sigma_{yy} + i\sigma_{xy}) = \gamma + \varepsilon \ln\left(\frac{r}{l_k}\right) \tag{5}$$

γ is argument of the complex stress intensity factor $K_I + iK_{II}$ which is defined as Fig. 2 and

$$\gamma = \mathrm{sign}\,(K_{II}) \cos^{-1}\left(\frac{K_I}{K_i}\right) \qquad (-\pi < \gamma < \pi), \tag{6}$$

$$\mathrm{sign}\,(K_{II}) = 1\ (K_{II} \geq 0), \quad = -1\ (K_{II} < 0), \tag{7}$$

where K_i is $K_i = \sqrt{K_I^2 + K_{II}^2}$. γ is easily transformed for another value of $l_k = l_k'$ [3].

$$\gamma' = \gamma + \varepsilon \ln (l_k'/l_k) \tag{8}$$

It is obvious by Eq. (5) that σ_{xy}/σ_{yy} on x-axis corresponds to K_{II}/K_I at $r = l_k$. In other words, K_{II}/K_I characterizes the ratio of shear stress and normal stress, σ_{xy}/σ_{yy}, at $r = l_k$. It is difficult to decide the suitable length of l_k. However, it is obvious that we should use a fixed value for l_k, because a set of K_I and K_{II} cannot express unique stress field around a crack tip for different lengths of l_k. Rice [4] recommended to select the length of l_k to be 1μm.

1.2 Crack Kinking Angle

He *et al.*[5, 6] proposed the maximum energy release rate criterion for an interface crack between dissimilar materials. They analytically obtained the kinking angle based on this model for an interface crack between dissimilar materials whose Dundurs parameter β in Eq. (2) is zero. The combination of materials whose β is zero is limited to the combination of similar materials. Geubelle and Knauss(8)

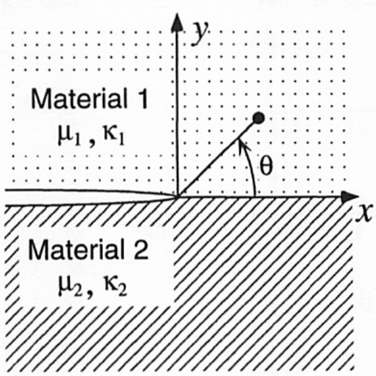

Fig.1 Coordinates system around an interface crack tip.

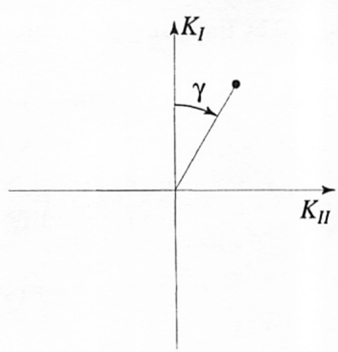

Fig. 2 Definition of γ on the K_I-K_{II} field.

applied the maximum energy release rate criterion to an interface crack between dissimilar materials whose β is not zero using the finite element method. They slightly extended the crack with different kinking angle, and obtained the energy release for each angle. The crack kinking angle expected by this theory depends on the crack extension length.

On the other hand, Yuki and Xu [8] proposed the maximum hoop stress criterion. The distribution of hoop stress $\sigma_{\theta\theta}$ around an interface crack as shown in Fig. 1

$$\sigma_{\theta\theta j} = \frac{\sqrt{K_I^2 + K_{II}^2}}{2\sqrt{2\pi r}\cosh(\varepsilon\pi)}\left[B\cos\psi - C\sin\psi\right] \quad (j = 1, 2) \tag{9}$$

$$\psi = \varepsilon\ln\left(r/l_k\right) \tag{10}$$

where $\sigma_{\theta\theta j}$ is hoop stress in the area of material j, the functions B and C are given as

$$B(\theta,\varepsilon,\gamma) = W_j\left[2\cos\left(\frac{\theta}{2}+\gamma\right) - \left(\cos\theta + 2\varepsilon\sin\theta\right)\cos\left(\frac{\theta}{2}-\gamma\right)\right] + \frac{1}{W_j}\cos\left(\frac{3}{2}\theta+\gamma\right) \tag{11}$$

$$C(\theta,\varepsilon,\gamma) = W_j\left[2\sin\left(\frac{\theta}{2}+\gamma\right) + \left(\cos\theta + 2\varepsilon\sin\theta\right)\sin\left(\frac{\theta}{2}-\gamma\right)\right] + \frac{1}{W_j}\sin\left(\frac{3}{2}\theta+\gamma\right) \tag{12}$$

$$W_1 = e^{-\varepsilon(\pi-\theta)}, \qquad W_2 = e^{\varepsilon(\pi-\theta)} \tag{13}$$

They simplified Eq. (9) by assumming Ψ=0 because of the small value of ε.

$$\sigma_{\theta\theta j} = \frac{\sqrt{K_I^2 + K_{II}^2}}{2\sqrt{2\pi r}\cosh(\varepsilon\pi)}B(\theta,\varepsilon,\gamma) \tag{14}$$

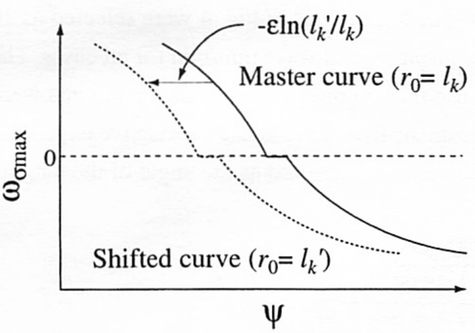

Fig. 3 Shift of the estimated kink angle with the value of r_0.

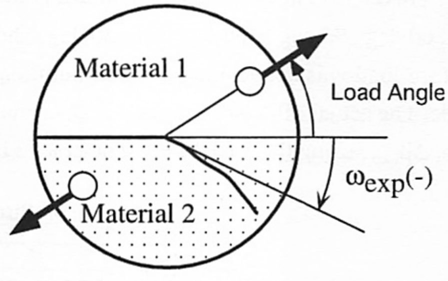

Fig. 4 Round shape mixed mode interface crack specimen and kink angle. (t= 12mm for case 1, 13mm for case 2 and 3).

$\sigma_{\theta\theta j}$ takes maximum value if $\partial B / \partial\theta = 0$. However, the value of Ψ in Eq. (10) cannot be ignored even if ε is small, because the absolute value of $\ln(r/l_k)$ increases with decreasing r. For example, if $\varepsilon = -0.033$ and $r/l_k = 0.001$, $\cos\Psi$ and $\sin\Psi$ are equal to 0.974 and 0.228 respectively.

We modified maximum hoop stress criterion proposed by Yuuki and Xu [8]. An interface crack kinks out in the direction of maximum hoop stress at some fixed distance, r_0, from a crack tip. The direction of maximum hoop stress can be obtained easily if $r_0 = l_k$ because Ψ takes zero at $r = l_k$. According to this theory, the expected crack kinking angle $\omega_{\sigma max}$ for $r_0 = l_k$ can be illustrated as 'master curve' in Fig. 3. The angle $\omega_{\sigma max}$ for another r_0, $r_0 = l_k'$, can be obtained by shifting the master curve by $\Delta\gamma = -\varepsilon \ln(l_k'/l_k)$ according to Eq. (8) as Fig. 3.

2. MIXED MODE FRACTURE TESTS OF INTERFACE CRACKS

Fracture tests were performed using round shapespecimens with mixed mode interface crack as shown in Fig. 4. Two combinations of materials, Aluminum - Epoxy resin A (Combination 1) and Aluminum - Epoxy resin B (Combination 2) were used for these specimens. Material properties of Aluminum, Epoxy resin A and Epoxy resin B are given in Table 1. A semicircular aluminum plate was set in a circular shape cast, then epoxy resin was pored into the cast. The cast was kept at 120℃ for 16 hours to cure the resin. The splicing surface of an aluminum plate was roughened by a piece of emery paper beforehand and release agent was applied on a part of the surface for making an interface crack.

Residual stress along a jointed interface is sometimes very great. We evaluated the residual stress by measuring the released strain, $\Delta\varepsilon_r$, of the end notched joint specimen at the delamination. The released strain, $\Delta\varepsilon_r$, was measured by a strain gauge attached to the surface of a specimen as shown in Fig. 5. The relative expansion ratio, $\Delta\beta_{exp.}$, between material 1 and 2 is expected by the beam theory. The SIF's of a round shape specimen caused by residual stress were calculated from $\Delta\beta_{exp.}$ by the virtual crack extension method (VCEM) with the finite element method (FEM) which developed in the previous study [9]. They are indicated in Table 2 for Combination 1 and Combination 2.

Mixed mode fracture tests were performed using the round shape specimens as shown in Fig. 4 and a universal testing machine (Shimazu Autograph). The load angles in Fig. 4 were selected as 30 deg., 60 deg., 90 deg., 120deg. and 150 deg. The rate of displacement was 1mm/min for all cases. The fracture load was determined as maximum load because the fracture modes of all specimens were brittle. The actual SIF's at the fracture can be obtained from the fracture load and the relative expansion ratio, $\Delta\beta_{exp.}$, using the VCEM [9]. The crack kinking angle was measured as the angle of the tangent

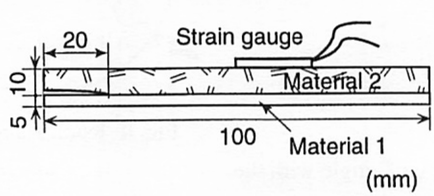

Fig. 5 End notched joint specimen for measuring residual stress.

Table 1 Material constants.

Material	Young's Modulus (GPa)	Poisson's Ratio
Aluminum	73.1	0.32
Resin A	3.35	0.43
Resin B	3.84	0.37

Table 2 Measured released strain and stress intensity factors caused by residual stress (l_k= 10 μm).

Case	$\Delta\varepsilon_r$ (μstrain)	$\Delta\beta_{exp}$ (%)	K_I (MPa$\sqrt{m}$)	K_{II} (MPa$\sqrt{m}$)
Comb. 1	961	-0.105	0.214	-0.450
Comb. 2	381	-0.0421	0.117	-0.176

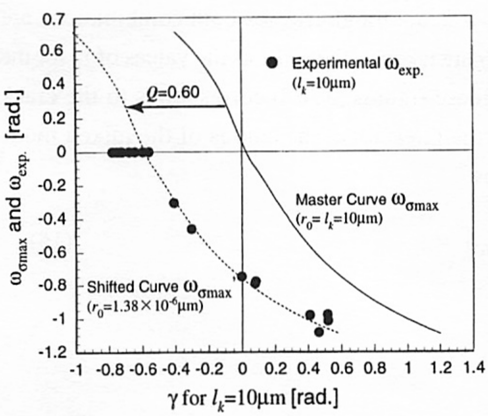

Fig. 6 Angle of crack kinking for Case 1.

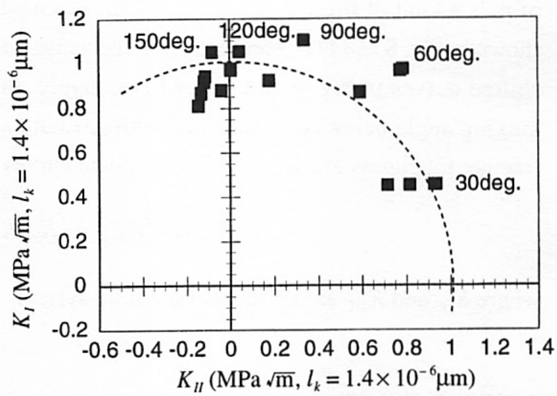

Fig. 8 Mixed mode fracture toughness for Case 1 (l_k= 1.4×10^{-6}μm).

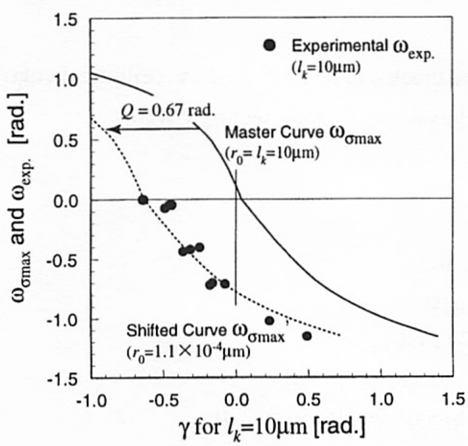

Fig. 7 Angle of crack kinking for Case 2.

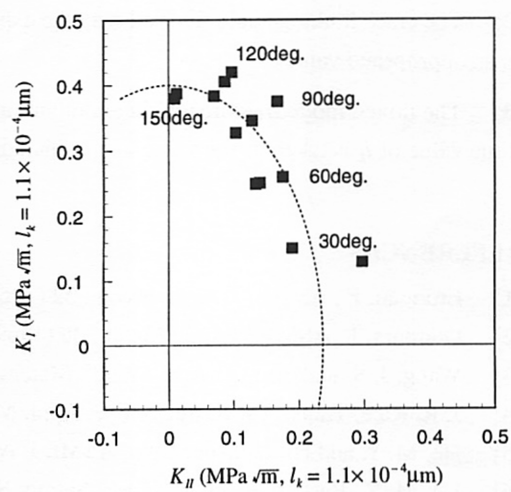

Fig. 9 Mixed mode fracture toughness for Case 2 (l_k= 1.1×10^{-4}μm).

of an extended crack at the initial crack tip in Fig. 4. The crack kinking angles measured on the both sides of a specimen were averaged.

The measured crack kinking angles with the master curves and the shifted curves of expected crack kinking angles for the combination 1 and 2 are shown in Fig. 6 and Fig. 7 respectively. The master curves do not correspond well with the experimental data in both cases. However, each shifted curve fits the experimental data very well. The values of r_0 at which the shifted curves fit the experimental data best are 1.4×10^{-6} μm and 1.1×10^{-4} μm respectively. These values of r_0 for both combinations seem to be too small as real distances which characterize the crack kinking. We consider that the value of r_0 is a kind of fitting parameter. The mixed mode fracture toughness for both combinations are shown in Fig. 8 and Fig. 9 respectively. The values of l_k are taken as the same as the values of r_0 for the shifted curves in Fig. 6 and Fig. 7 respectively. In these figures, K_{II}= 0 corresponds to the crack kinking angle being zero (the crack extended along the interface). The curves of the mixed mode fracture toughness are described by elliptical curves as

$$\left(K_I/K_{IC}\right)^2 + \left(K_{II}/K_{IIC}\right)^2 = 1 \tag{15}$$

where K_{IC} and K_{IIC} are constants for a joint system.

3. CONCLUSION

(1) The crack kinking angle of an interface crack between dissimilar materials is well described by the modified maximum hoop stress criterion.

(2) The crack kinking angle fits well with the experimental data if the evaluation distance r_0 is taken as an appropriate value.

(3) The mixed mode fracture toughness of an interface crack can be described by a elliptical curve if the value of l_k is taken as the value of r_0 which describes the crack kinking angle well.

REFERENCES

[1] Erdogan, F., ASME J. Appl. Mech., **32** (1965) p. 403.

[2] Dundurs, J., ASME J. Appl. Mech., **36** (1969) p. 650.

[3] Wang, J. S. and Suo, Z., Acta Metall. Mater., **38**-7 (1990) p. 1279.

[4] J. R. Rice, Trans. ASME Series E, J. Appl. Mech., **55** (1988) p. 98.

[5] He, M. Y. and Hutchinson, J. W., ASME J. Appl. Mech., **56** (1989) p. 270.

[6] He, M. Y., Bartlett, A. *et al.*, J. Am. Seram. Soc., **74**-4 (1991) p. 767.

[7] Geubelle, P. H. and Knauss, W. G., ASME J. Appl. Mech., **61** (1994) p. 560.

[8] Yuuki, R. and Xu, J. Q., Eng. Frac. Mech., **41** (1992) p. 635.

[9] Ikeda, T., Komohara, Y. and Miyazaki, N., Advances in Electronic Packaging, ASME EEP **19**-2 (1998) p. 1137.